



# Viewing

## Part I (History and Overview of Projections)

# Lecture Topics

- ▶ History of Projection in Art
- ▶ Geometrical Constructions
- ▶ Types of Projection (Parallel and Perspective)

## Drawing as Projection (Turning 3D to 2D)

- ▶ Painting based on mythical tale as told by Pliny the Elder: Corinthian man traces shadow of departing lover
- ▶ Projection through use of shadows

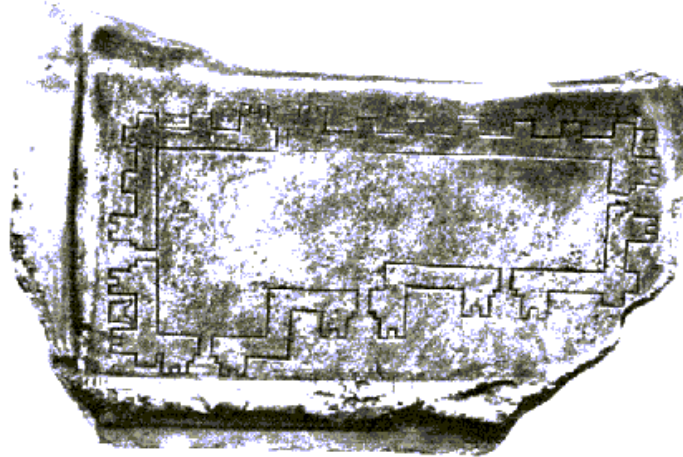
- ▶ Detail from: **The Invention of Drawing (1830)**

Karl Friedrich Schinkel  
(Mitchel p.1)

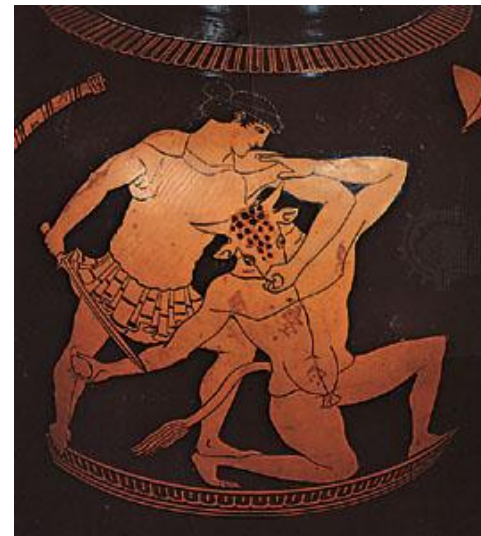


## Early Forms of Projection (1/2)

- ▶ Plan view (*parallel*, specifically orthographic, projection) from Mesopotamia (2150 BC): Earliest known technical drawing in existence
- ▶ Greek vase from the late 6<sup>th</sup> century BC: Shows signs of attempts at *perspective*!
  - ▶ Note relative sizes of thighs and lower legs of minotaur



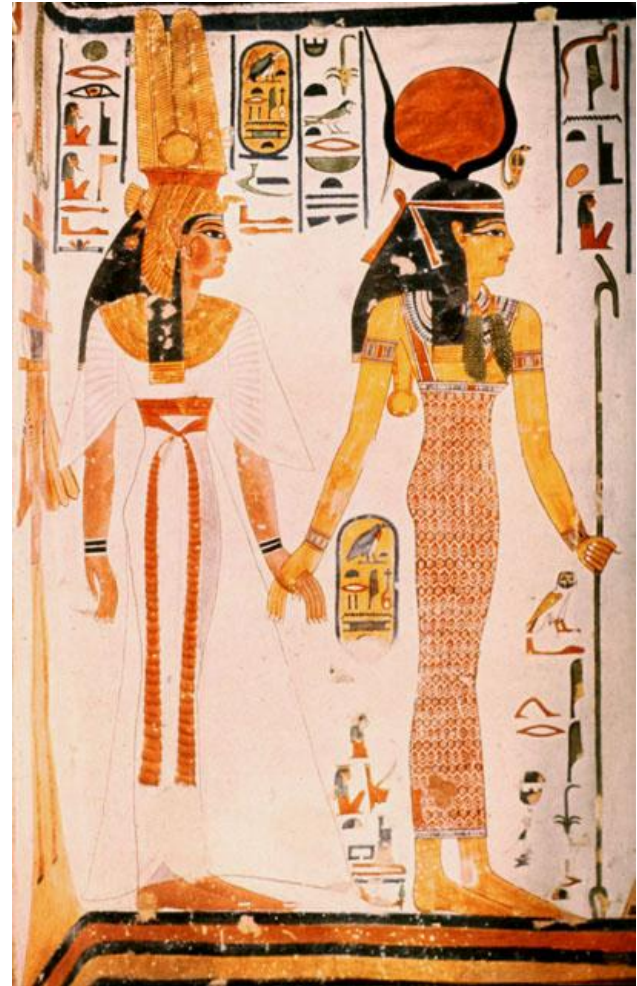
Carlbom Fig. 1-1



Theseus Killing the Minotaur by the Kleophrades Painter

## Early Forms of Projection (2/2)

- ▶ Ancient Egyptian Art:
  - ▶ Multiple Viewpoints
  - ▶ Parallel Projection
- ▶ Tomb of Nefertari, Thebes (19th Dyn, ~1270 BC), *Queen led by Isis*. Mural
- ▶ Note how the depiction of the body implies a front view but the feet and head imply side view (early cubism)

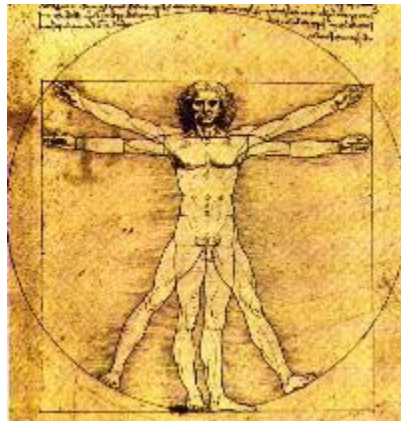




# The Renaissance

- ▶ Starting in the 13<sup>th</sup> century (AD): New emphasis on importance of individual viewpoint, world interpretation, power of observation (particularly of nature: astronomy, anatomy, etc)

- ▶ Massaccio
- ▶ Donatello
- ▶ Leonardo
- ▶ Newton



Ender, Tycho Brahe and Rudolph II in

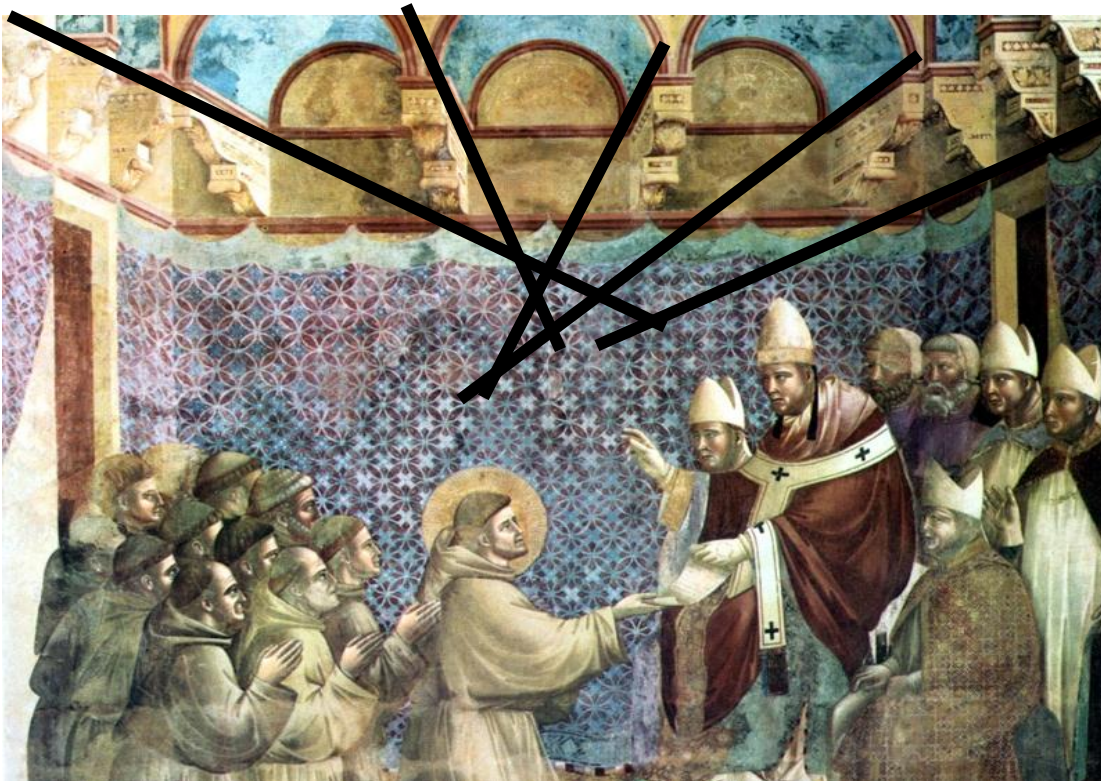
## Prague

(detail of clockwork), c. 1855

- ▶ Universe as clockwork: rebuilding the universe more systemically and mechanically

## Early Attempts at Perspective

- ▶ In art, an attempt to represent 3D space more realistically
- ▶ Earlier works invoke a sense of 3D space but not systematically
  - ▶ Lines converge, but no single vanishing point



Giotto

**Franciscan Rule  
Approved**

Assisi, Upper Basilica

c.1295-1300

## Brunelleschi and Vermeer

- ▶ Brunelleschi invented systematic method of determining perspective projections (early 1400's). He created demonstration panels with specific viewing constraints for complete accuracy of reproduction. Note the perspective is accurate only from one POV
- ▶ Vermeer created *perspective boxes* where picture, when viewed through viewing hole, had correct perspective

- ▶ Vermeer on the web:

- ▶ [http://www.grand-illusions.com/articles/mystery\\_in\\_the\\_mirror/](http://www.grand-illusions.com/articles/mystery_in_the_mirror/)
- ▶ <http://essentialvermeer.20m.com/>
- ▶ <http://brightbytes.com/cosite/what.html>



**Perspective Box**  
Samuel van Hoogstraten  
National Gallery, London

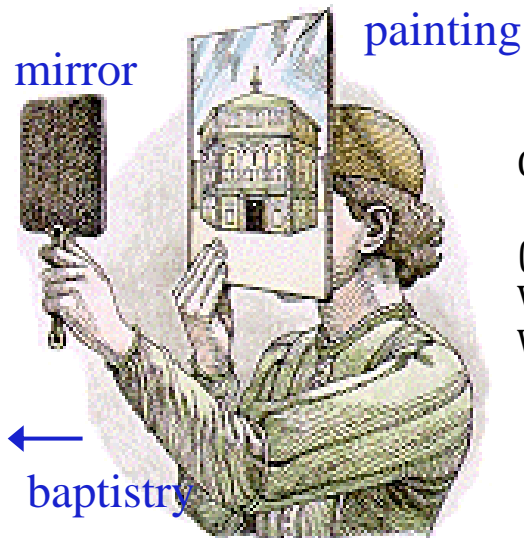


**Perspective Box of a Dutch Interior**  
Samuel van Hoogstraten  
National Gallery, London



## Brunelleschi's Method

- ▶ Brunelleschi was reported to have used this setup to determine the accuracy of his paintings
- ▶ The realism of his paintings are evidence that Brunelleschi had some systematic method for determining perspective projections, although the procedure he used was never documented
- ▶ His illusion inspired other artists to explore linear perspective

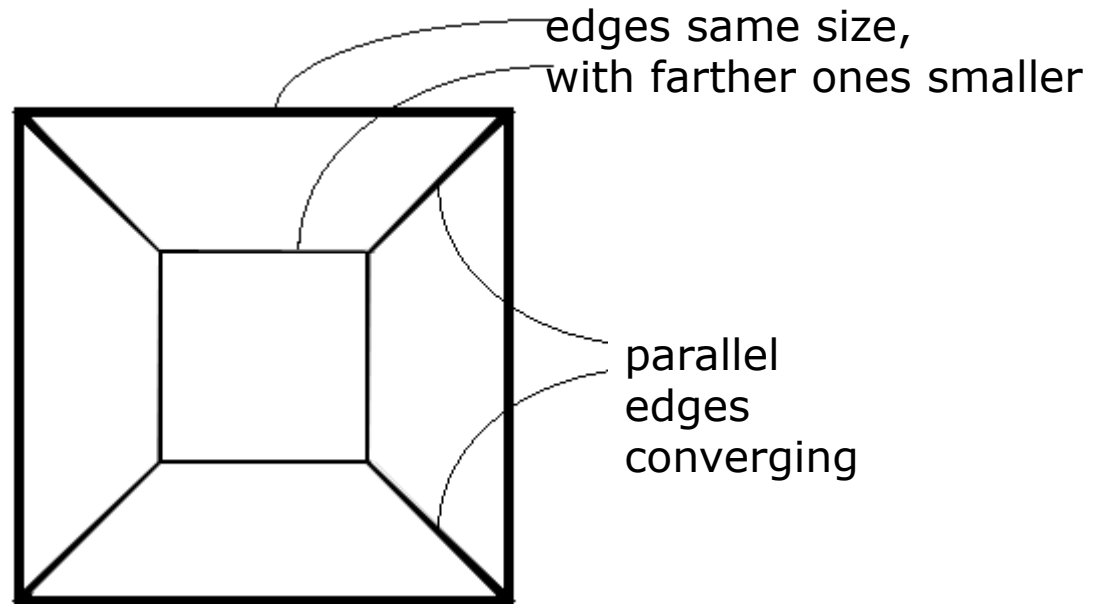


Credit to COGS011 for these images  
(Perception, Illusion and Visual Art, William Warren)



## Rules of Linear Perspective

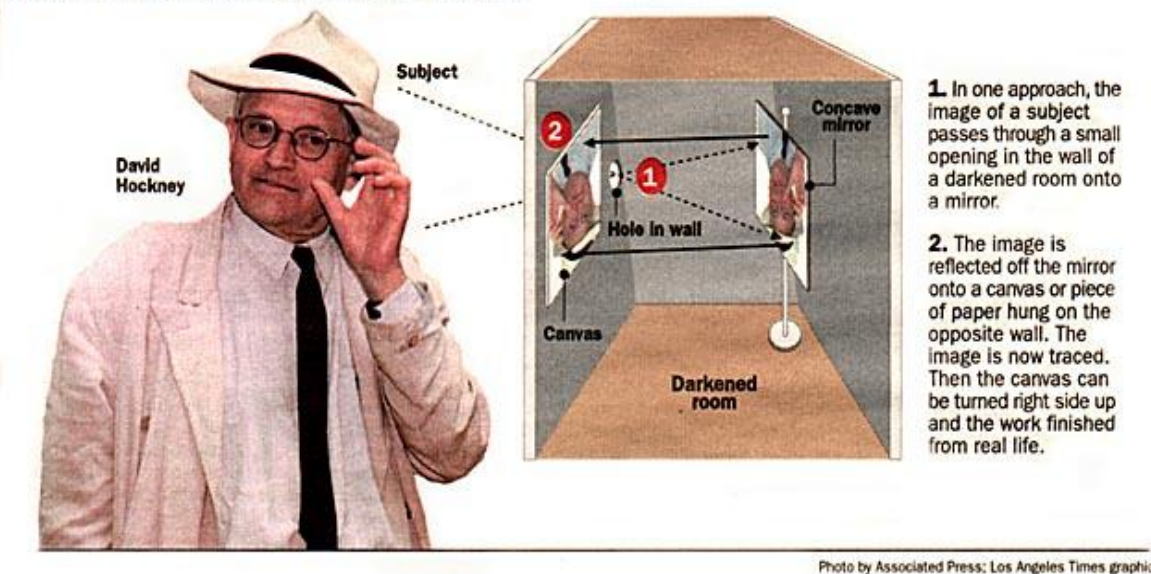
- ▶ Driving ideas behind Linear Perspective:
- ▶ Parallel lines converge (in 1, 2, or 3 axes) to *vanishing point*
- ▶ Objects farther away are more *foreshortened* (i.e., smaller) than closer ones
- ▶ Example: perspective cube



## A Similar Idea: Camera Obscura

- ▶ An artist named David Hockney proposed that many Renaissance artists, including Vermeer, might have been aided by *camera obscura* while painting their masterpieces, raising a big controversy

### How the Camera Obscura Works



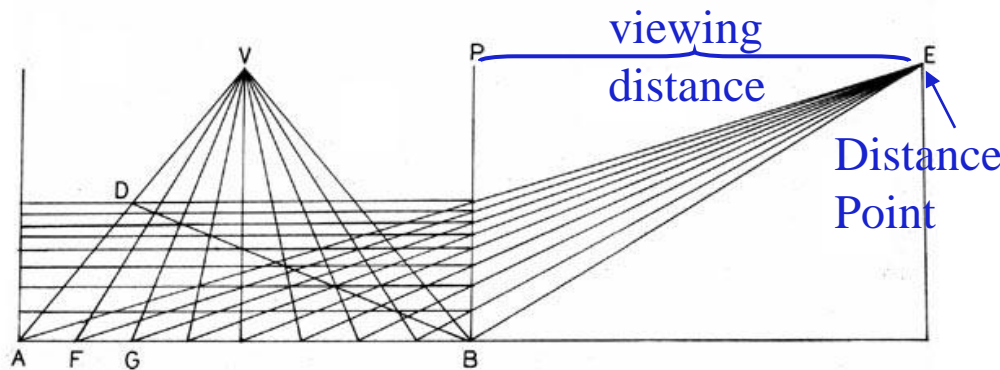
Hockney, D. (2001) *Secret Knowledge: Rediscovering the Lost Techniques of the Old Masters*.

New York: Viking Studio.  
Stork, D. (2004) Optics and Realism in Renaissance Art. *Scientific American* 12, 52-59.

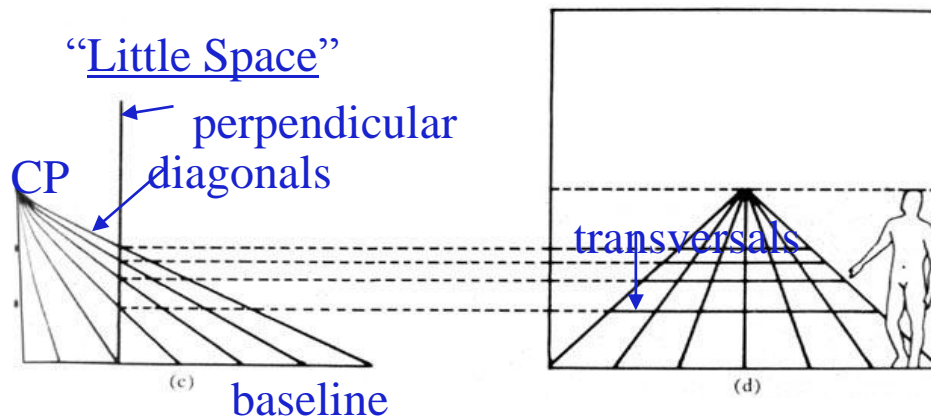
- ▶ David Stork, a Stanford optics expert, refuted Hockney's claim in the heated 2001 debate about the subject among artists, museum curators and scientists. He also wrote the article "Optics and Realism in Renaissance Art", using scientific techniques to disprove Hockney's theory

## Linear Perspective (Vanishing Points)

- Both Da Vinci and Alberti created accurate geometric ways of incorporating linear perspective into a drawing using the concept vanishing points



Da Vinci's Method



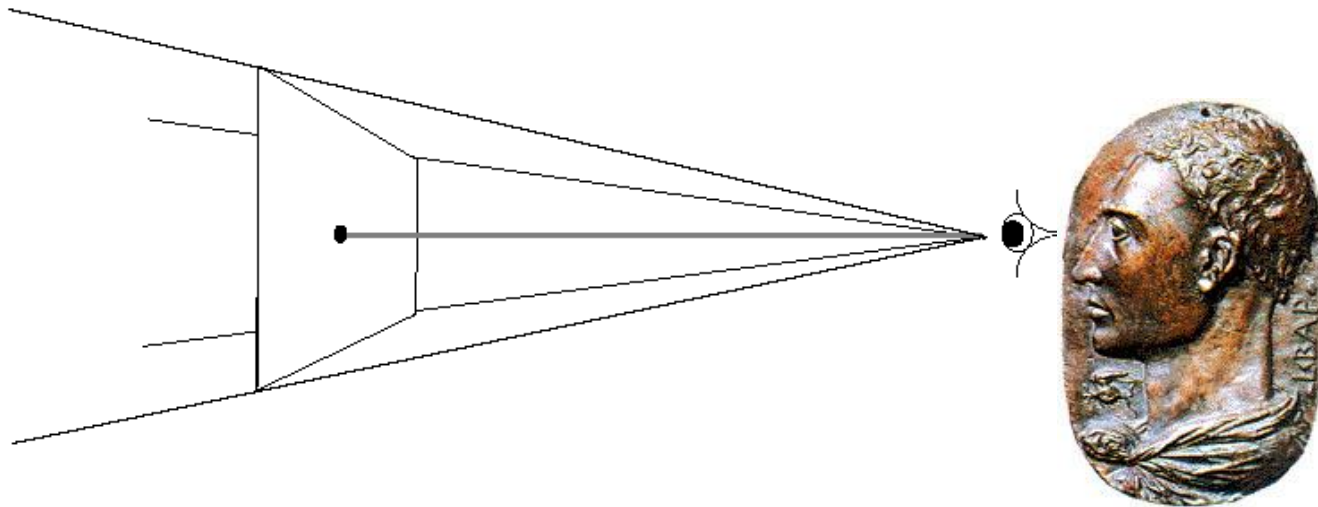
Alberti's Method

Credit to COGS011 for these images  
(Perception, Illusion and Visual Art,  
William Warren)



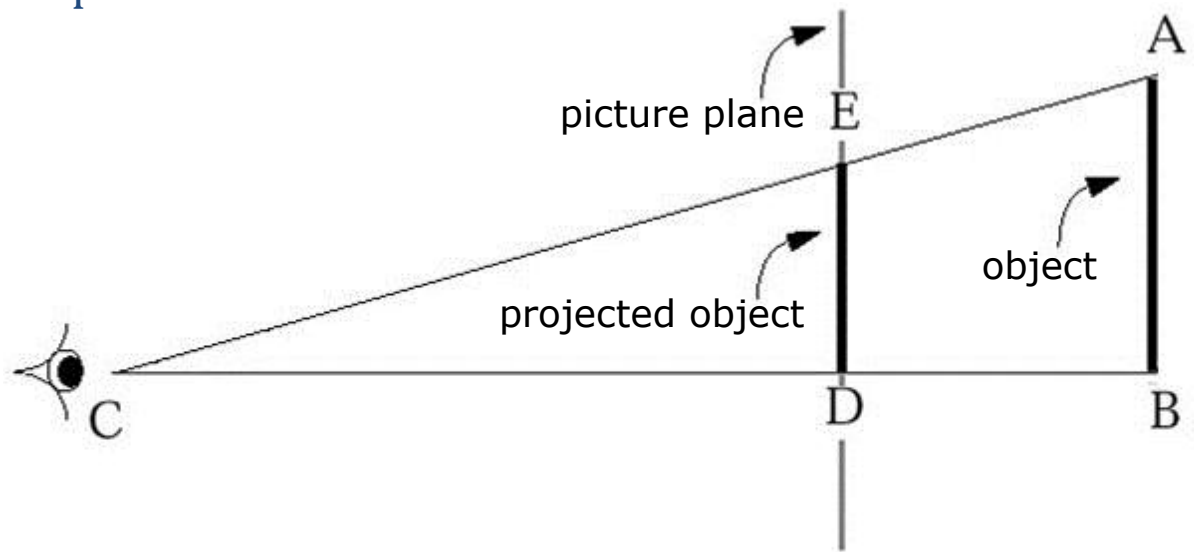
## Alberti on Linear Perspective (View Points)

- ▶ Published first treatise on perspective, *Della Pittura*, in 1435
- ▶ “A painting [**the projection plane**] is the intersection of a visual pyramid [**view volume**] at a given distance, with a fixed center [**center of projection**] and a defined position of light, represented by art with lines and colors on a given surface [**the rendering**].” (Leono Battista Alberti (1404-1472), *On Painting*, pp. 32-33)
- ▶ A different way of thinking about perspective from the vanishing point



## Triangles and Geometry (1/2)

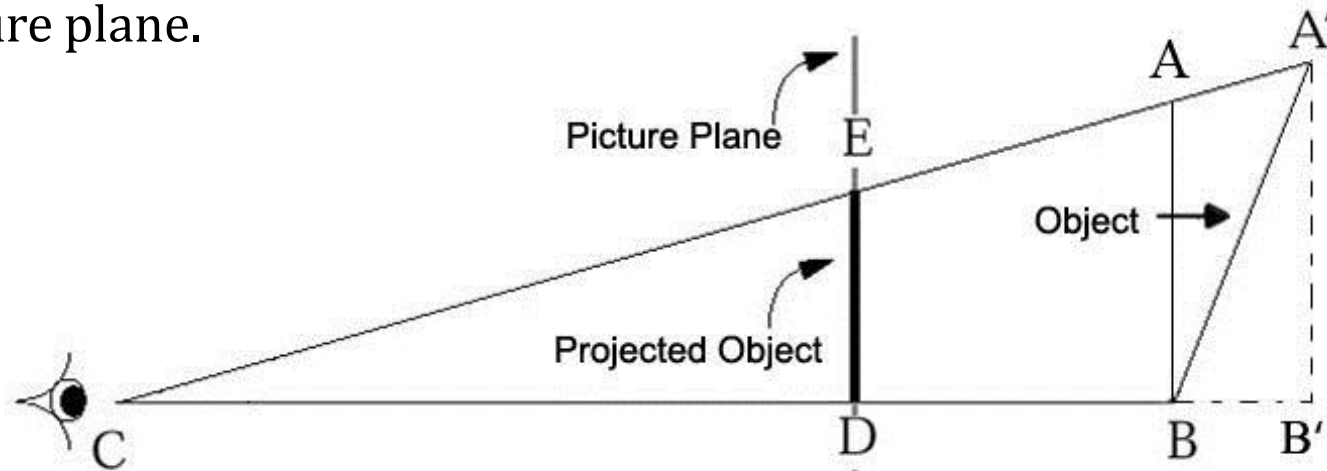
- ▶ Idea of “visual pyramid” implies use of triangular geometry
- ▶ Easy to project object on to an image plane based on
  - ▶ height of object (AB)
  - ▶ distance from eye to object (CB)
  - ▶ distance from eye to picture (projection) plane (CD)
  - ▶ and using relationship  $CB : CD$  as  $AB : ED$



$$CB : CD \text{ as } AB : ED$$

## Triangles and Geometry (2/2)

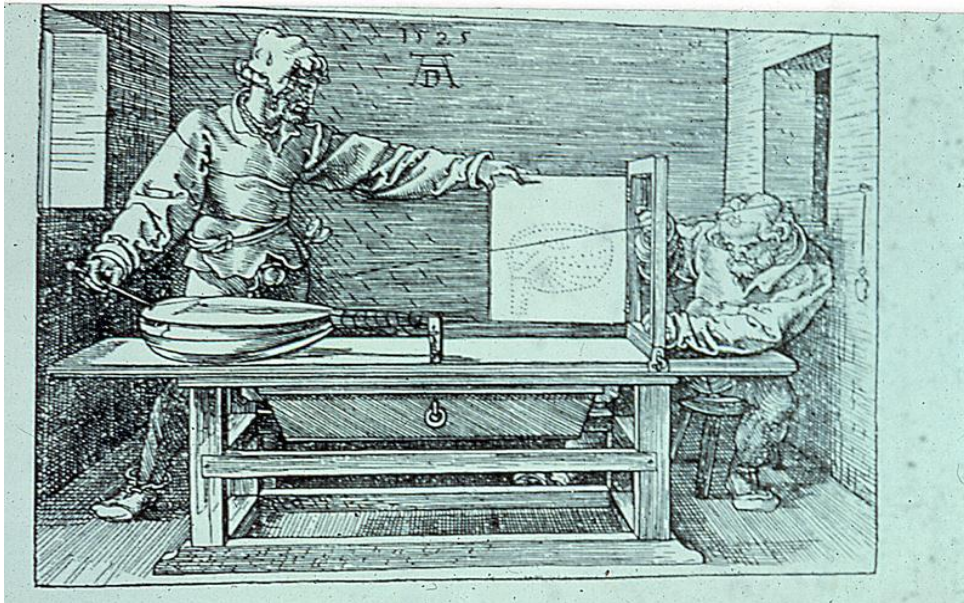
- ▶ The general case: the object we're considering is not parallel to the picture plane.



- ▶ Find the projection ( $B'$ ) of ( $A'$ ) onto the line  $CB$ 
  - ▶ The dot product of vector  $\mathbf{a}$  with vector  $\mathbf{b}$  is the projection of  $\mathbf{a}$  onto  $\mathbf{b}$
  - ▶  $CB' = \text{dot}(CA', \text{normalize}(CB)) * \text{normalize}(CB)$
- ▶  $CB':CD$  as  $A'B':ED$

## Dürer Woodcut

- ▶ Concept of similar triangles described both geometrically and mechanically in widely read treatise by Albrecht Dürer (1471-1528)
- ▶ Refer to chapter 3 of the book for more details



Albrecht Dürer

### Artist Drawing a Lute

Woodcut from Dürer's work about the Art of Measurement.  
'Underweysung der messung',  
Nuremberg, 1525



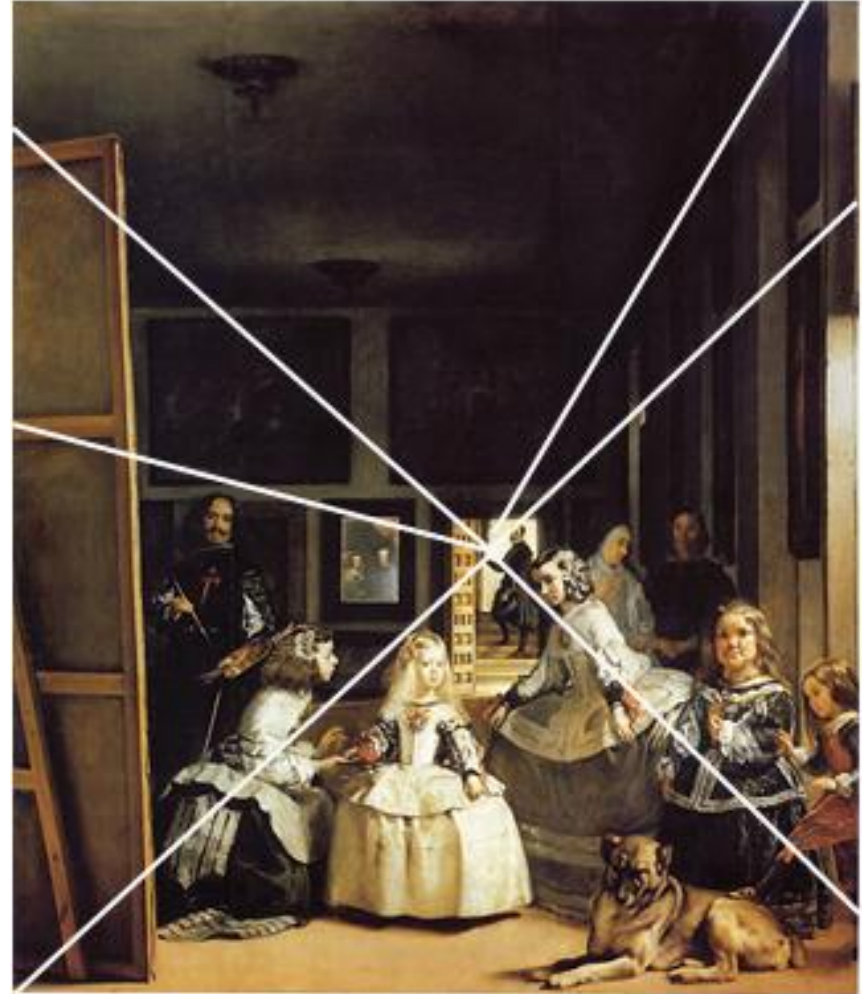
## Art of Perspective(1/5)



Robert Campin - The Annunciation Triptych (ca. 1425)

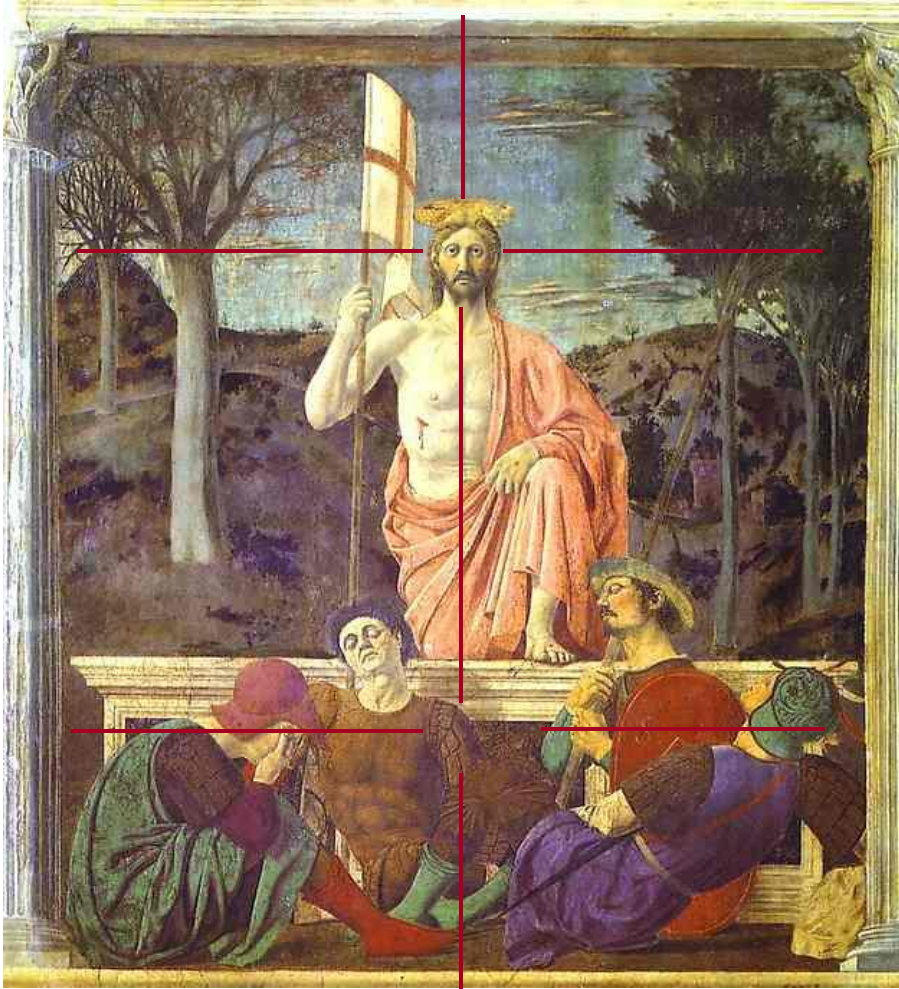
## Art of Perspective(2/5)

- ▶ Point of view influences content and meaning of what is seen
- ▶ Are royal couple in mirror about to enter room? Or is their image a reflection of painting on far left?
- ▶ Analysis through computer reconstruction of the painted space: royal couple in mirror is reflection from canvas in foreground, not reflection of actual people (Kemp pp. 105-108)





## Art of Perspective(3/5)

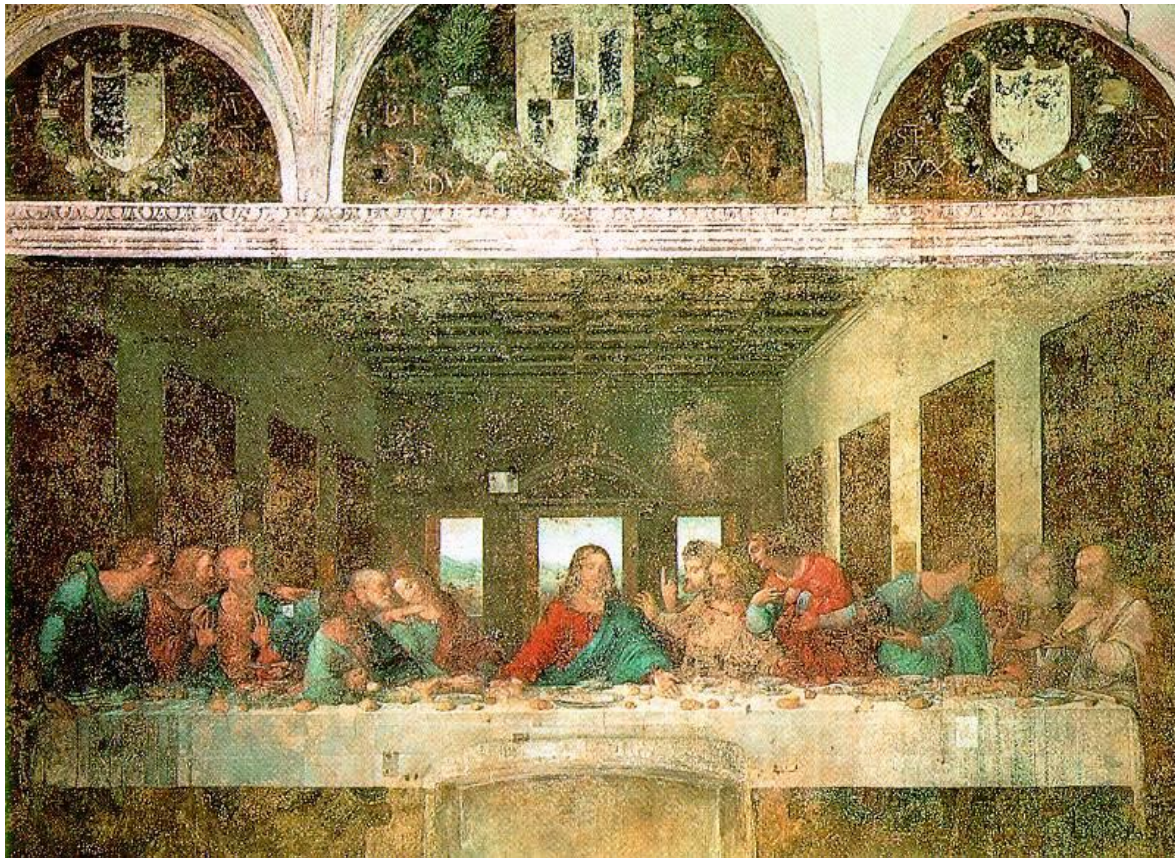


- Perspective can be used in unnatural ways to control perception
- Use of two viewpoints concentrates viewer's attention alternately on Christ and sarcophagus

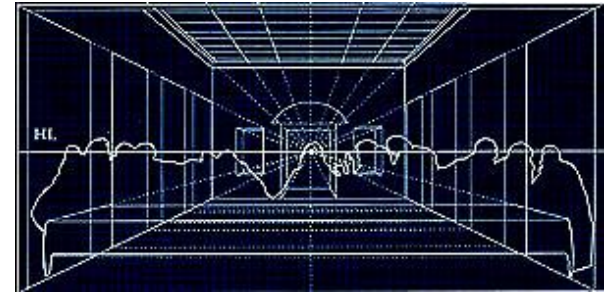
Piero della Francesca The Resurrection (1460)



## Art of Perspective(4/5)

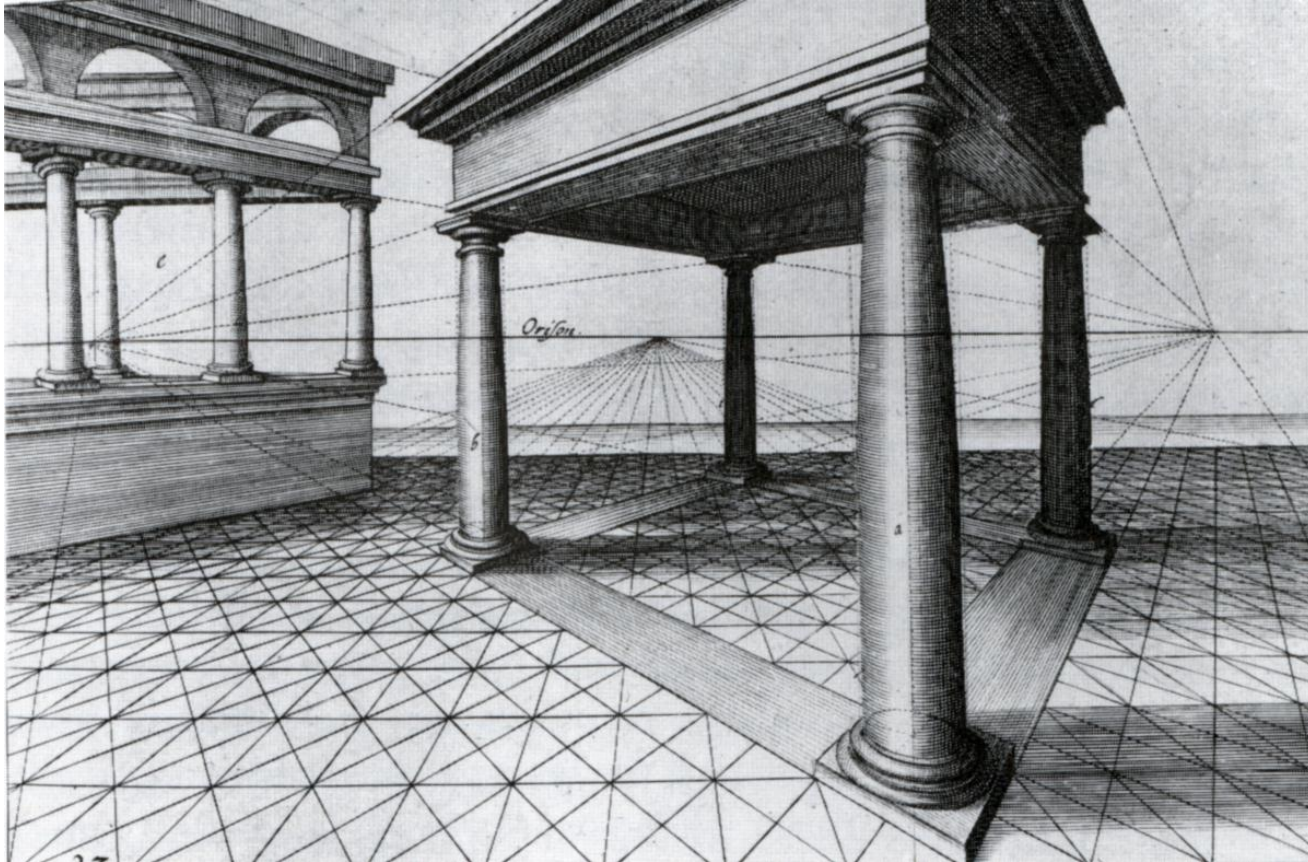


Leonardo da Vinci The Last Supper (1495)





## Art of Perspective(5/5)

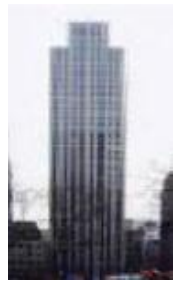
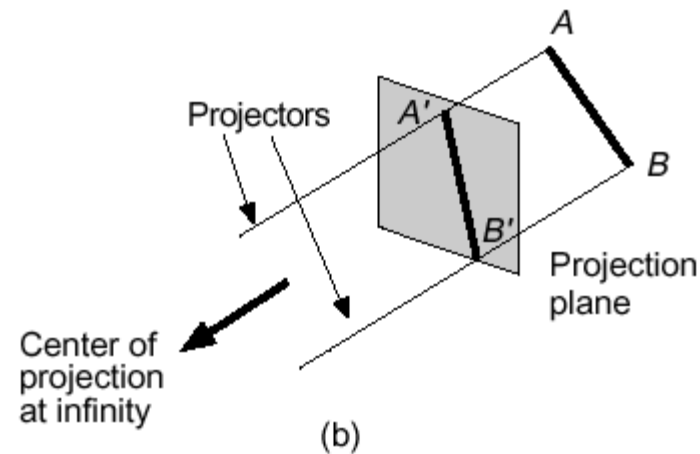
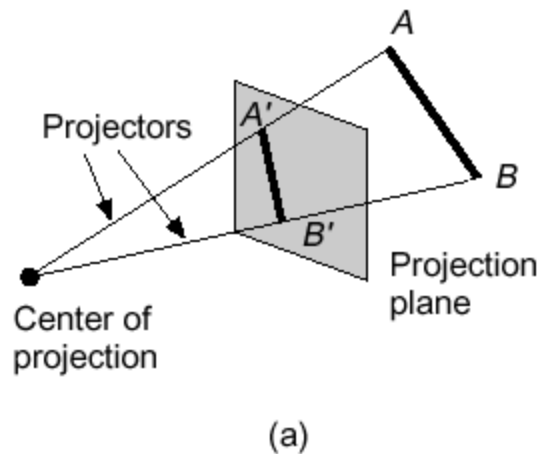


Vredeman de Vries's **Perspective**, Kemp p.117

Two Vanishing Points, Two point perspective

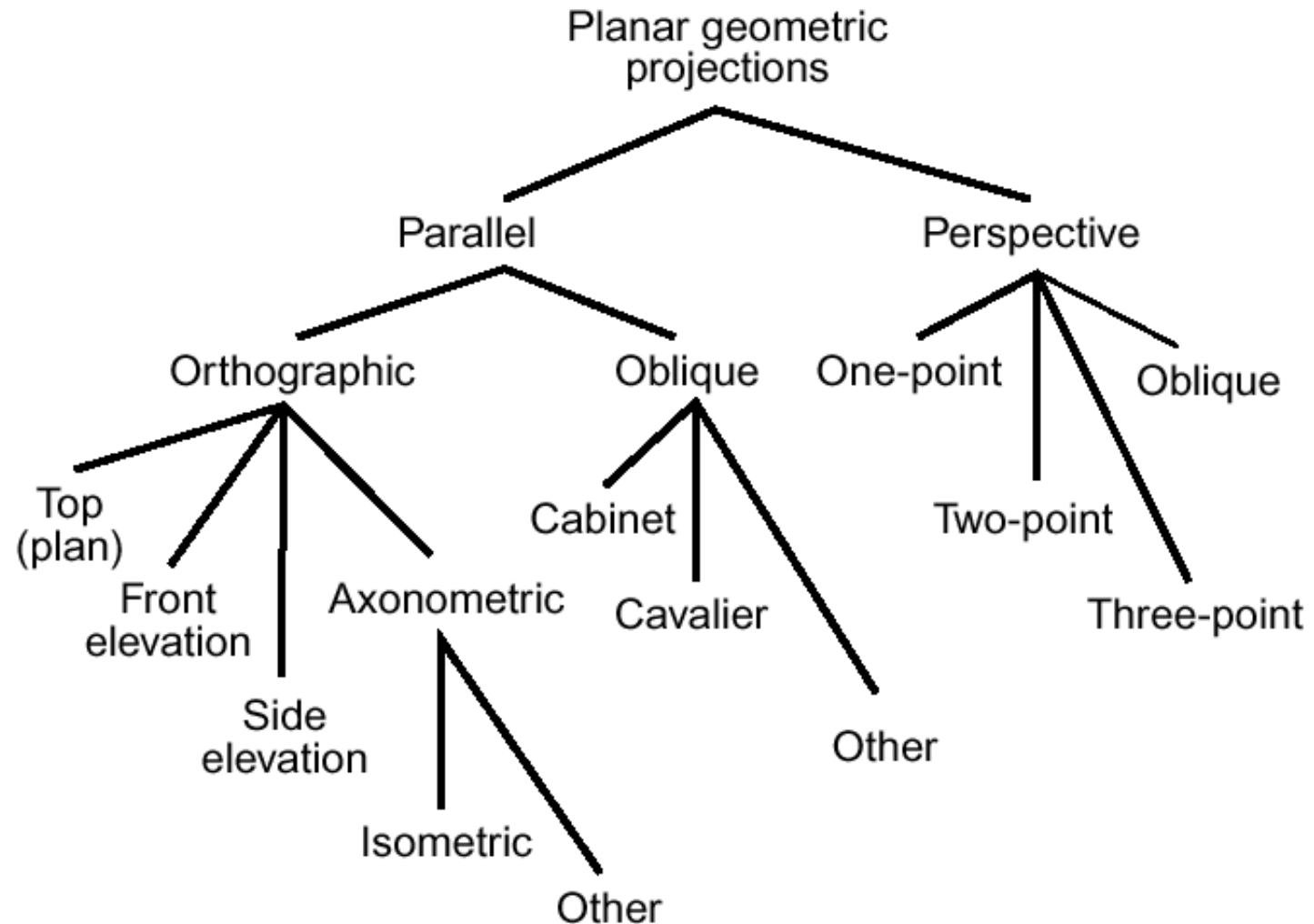
# Main Classes of Planar Geometrical Projections

- ▶ Perspective: determined by Center of Projection (COP) (in our diagrams, the “eye”) (a)
  - ▶ More natural, simulates what our eyes or a camera sees
- ▶ Parallel: determined by Direction of Projection (DOP) (projectors are parallel—do not converge to “eye” or COP). Alternatively, COP is at  $\infty$  (b)
  - ▶ Used in engineering and architecture for measurement purposes



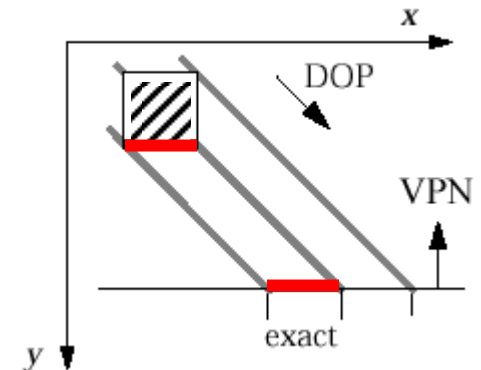
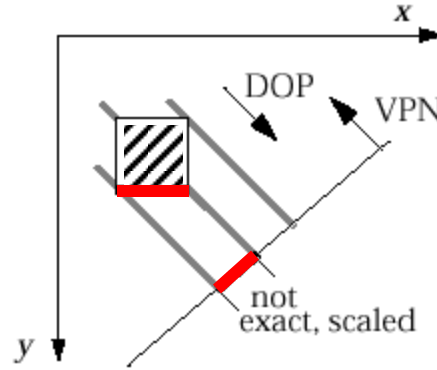
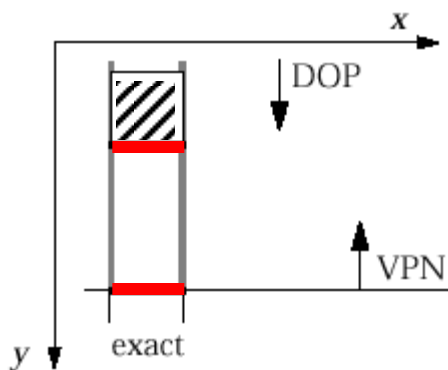
- ▶ In general, a projection is determined by where you place the projection plane relative to principal axes of object (relative angle and position), and what angle the projectors make with the projection plane

# Logical Relationship Between Types of Projections



# Overview of Parallel Projections

- Assume object face of interest lies in principal plane, i.e., parallel to  $xy$ ,  $yz$ , or  $zx$  planes. (DOP = Direction of Projection, VPN = View Plane Normal)



## 1) Multiview Orthographic

- VPN || a principal coordinate axis
- DOP || VPN
- shows single face, exact measurements

## 2) Axonometric

- NOT** (VPN || a principal) coordinate axis
- DOP || VPN
- adjacent faces, none exact, uniformly foreshortened (function of angle between face normal and DOP)

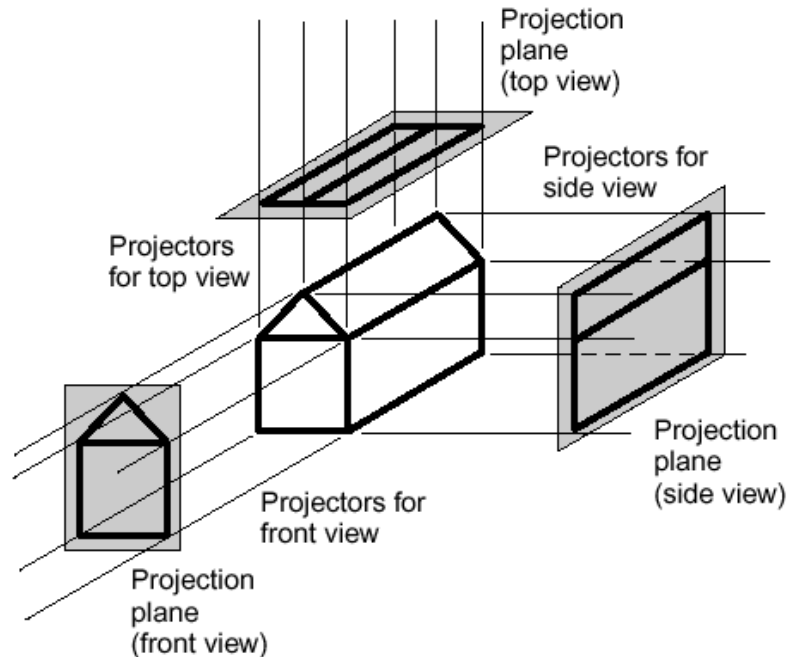
## 3) Oblique

- NOT** (VPN || a principal) coordinate axis
- DOP || VPN
- adjacent faces, one exact, others uniformly foreshortened



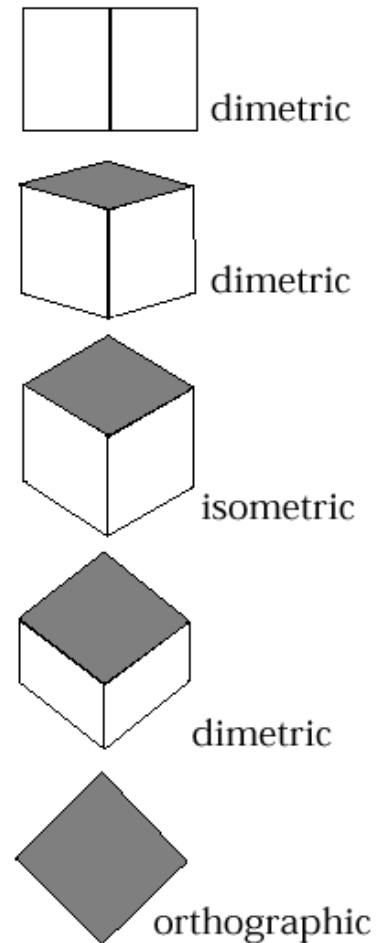
# MultiView Orthographic (Parallel)

- ▶ Used for:
  - ▶ engineering drawings of machines, machine parts
  - ▶ working architectural drawings
- ▶ Pros:
  - ▶ accurate measurement possible
  - ▶ all views are at same scale
- ▶ Cons:
  - ▶ does not provide “realistic” view or sense of 3D form
- ▶ Usually need multiple views to get a three-dimensional feeling for object



## Axonometric (Parallel)

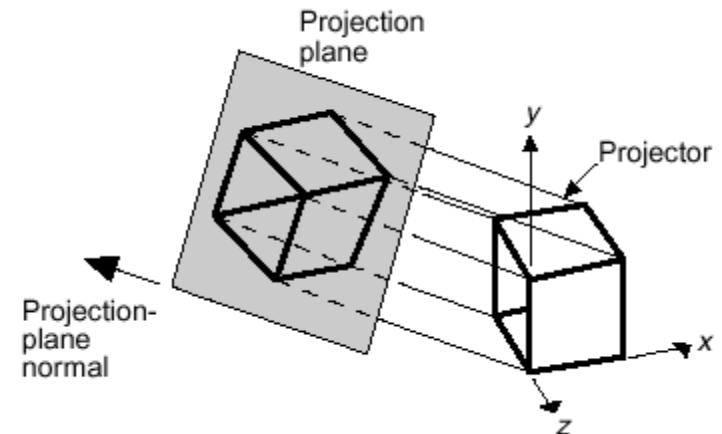
- ▶ Same method as multiview orthographic projections, except projection plane not parallel to any of **coordinate planes**; parallel lines equally foreshortened
- ▶ **Isometric**: Angles between all three principal axes equal ( $120^\circ$ ). Same scale ratio applies along each axis
- ▶ **Dimetric**: Angles between two of the principal axes equal; need two scale ratios
- ▶ **Trimetric**: Angles different between three principal axes; need three scale ratios



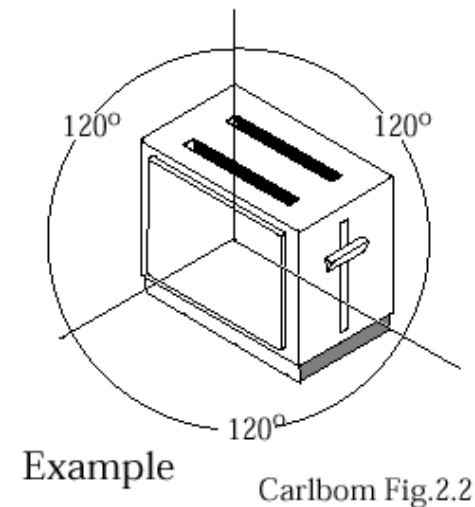
Carlbon Fig.3-8

# Isometric Projection

- ▶ Used for:
  - ▶ catalogue illustrations
  - ▶ patent office records
  - ▶ furniture design
  - ▶ structural design
  - ▶ 3d Modeling in real time (Maya, AutoCad, etc.)
- ▶ Pros:
  - ▶ don't need multiple views
  - ▶ illustrates 3D nature of object
  - ▶ measurements can be made to scale along principal axes
- ▶ Cons:
  - ▶ lack of foreshortening creates distorted appearance
  - ▶ more useful for rectangular than curved shapes

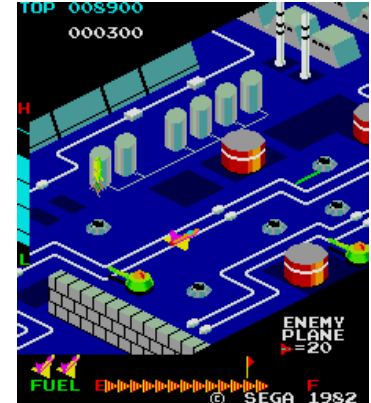
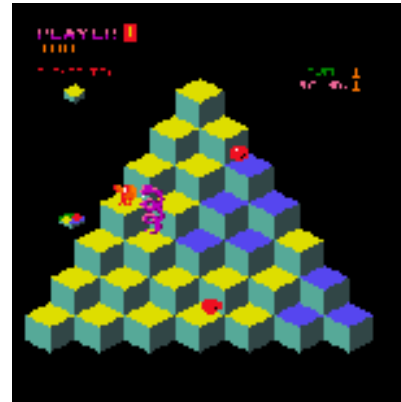


Construction of an isometric projection:  
projection plane cuts each principal axis by  $45^\circ$



# Axonometric Projection in Games

- ▶ Video games have been using isometric projection for ages.
  - ▶ It all started in 1982 with *Q\*Bert* and *Zaxxon* which were made possible by advances in raster graphics hardware
- ▶ Still in use today when you want to see things in distance as well as things close up (e.g. strategy, simulation games)
  - ▶ *SimCity IV*
  - ▶ *StarCraft II*
- ▶ While many games technically use axonometric views, the general style is still referred to isometric or inappropriately, “2.5D”/ “three quarter”.



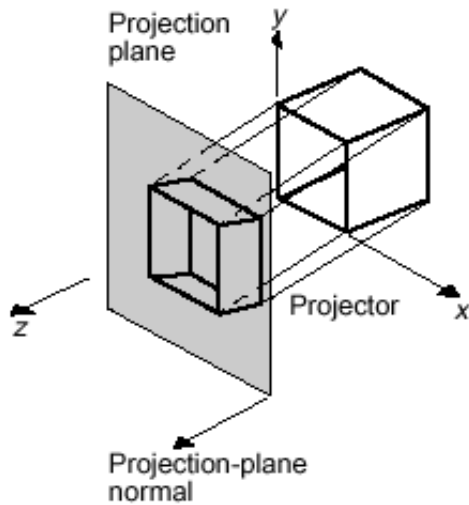
## Oblique Projection (Parallel)

- ▶ Projectors at oblique angle to projection plane; view cameras have accordion housing, can adjust the projection plane
- ▶ Pros:
  - ▶ can present exact shape of one face of an object (can take accurate measurements): better for elliptical shapes than axonometric projections, better for “mechanical” viewing
  - ▶ lack of perspective foreshortening makes comparison of sizes easier
  - ▶ displays some of object’s 3D appearance
- ▶ Cons:
  - ▶ objects can look distorted if careful choice not made about position of projection plane (e.g., circles become ellipses)
  - ▶ lack of foreshortening (not realistic looking)

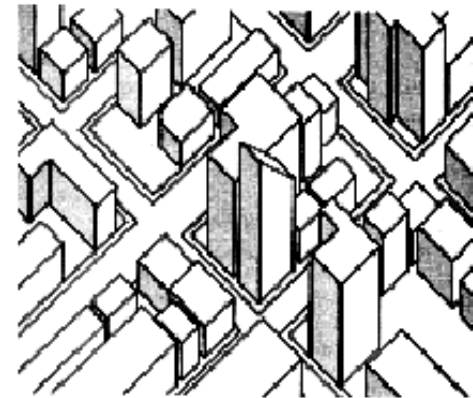




## Examples of Oblique Projections

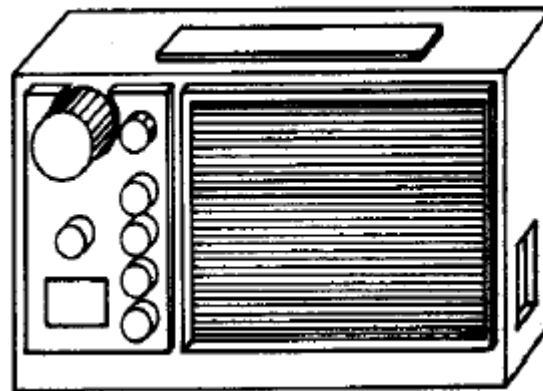


Construction of oblique parallel projection



(Carlbon Fig. 2-6)

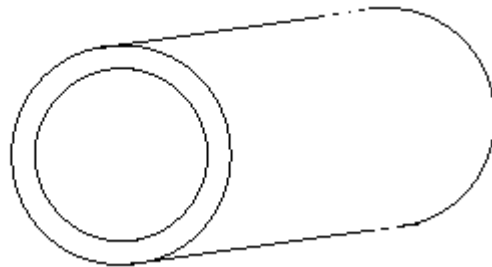
Plan oblique projection of city



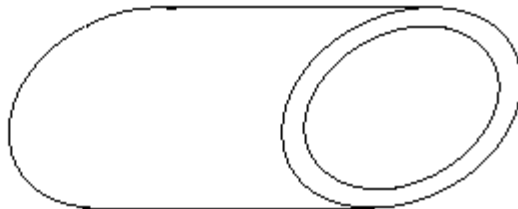
Front oblique projection of radio (Carlbon Fig. 2-4)

## Rules for Constructing Oblique Views

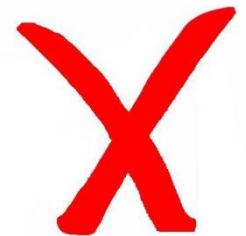
- ▶ Rules for placing projection plane for oblique views: projection plane should be chosen according to one or several of following:
  - ▶ parallel to most irregular of principal faces, or to one which contains circular or curved surfaces
  - ▶ parallel to longest principal face of object
  - ▶ parallel to face of interest



Projection plane  
parallel to circular face

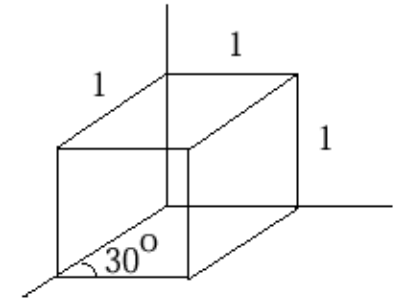
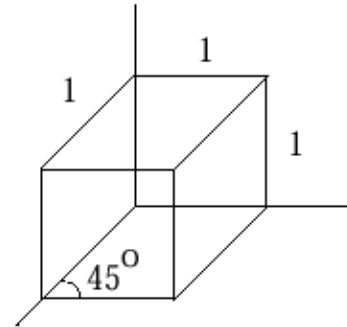
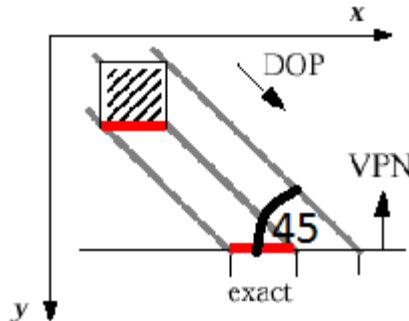


Projection plane not  
parallel to circular face

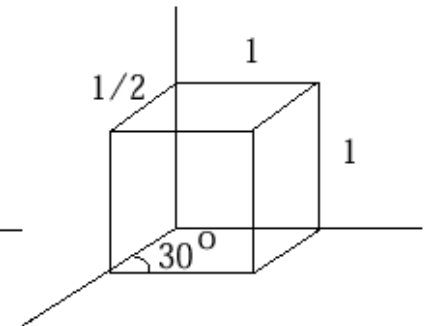
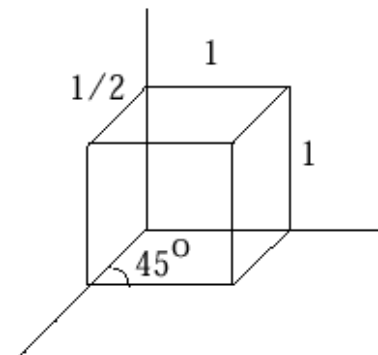
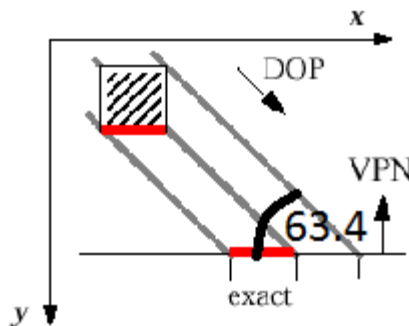


# Main Types of Oblique Projections

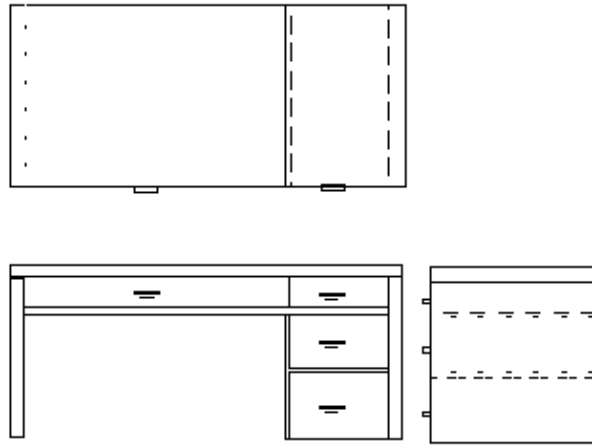
- ▶ *Cavalier*: Angle between projectors and projection plane is  $45^\circ$ . Perpendicular faces projected at full



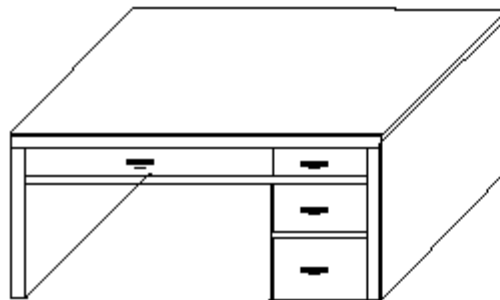
- ▶ *Cabinet*: Angle between projectors and projection plane:  $\arctan(2) = 63.4^\circ$ . Perpendicular faces projected at 50% scale



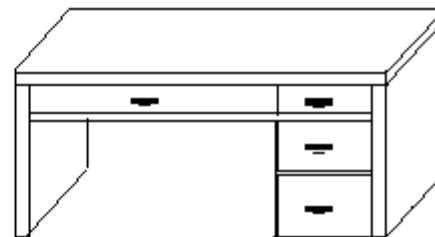
## A Desk in Parallel



multiview orthographic



cavalier



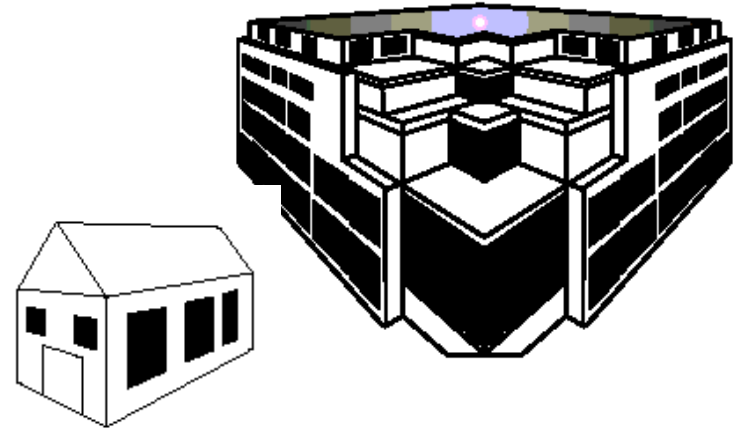
cabinet

Carlbon Fig. 3-2



# Perspective Projections

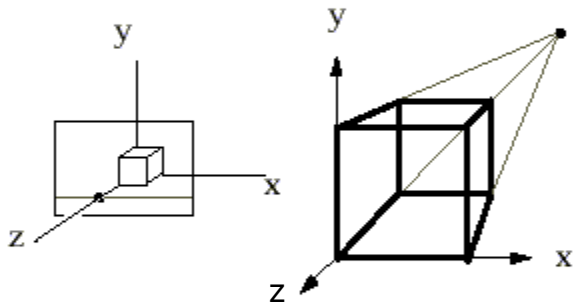
- ▶ Used for:
  - ▶ fine art
  - ▶ Human visual system...
- ▶ Pros:
  - ▶ gives a realistic view and feeling for 3D form of object
- ▶ Cons:
  - ▶ does not preserve shape of object or scale (except where object intersects projection plane)
- ▶ Different from a parallel projection because
  - ▶ parallel lines not parallel to the projection plane converge
  - ▶ size of object is diminished with distance
  - ▶ foreshortening is not uniform
- ▶ Two understandings: **Vanishing Point and View Point**
- ▶ There are also oblique perspective projections (same idea as parallel oblique), we'll see an example next lecture



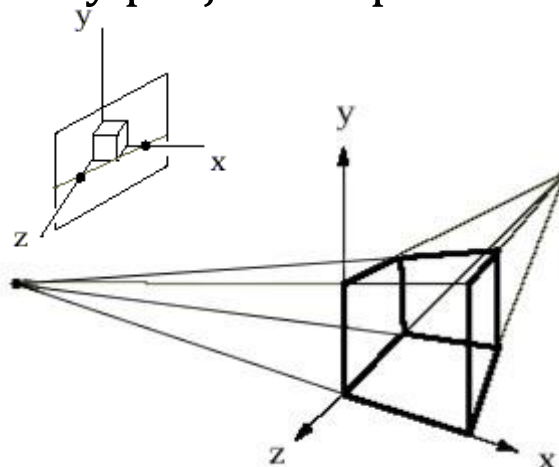
If we were viewing this scene using parallel projection, the tracks would not converge

## Vanishing Points (1/2)

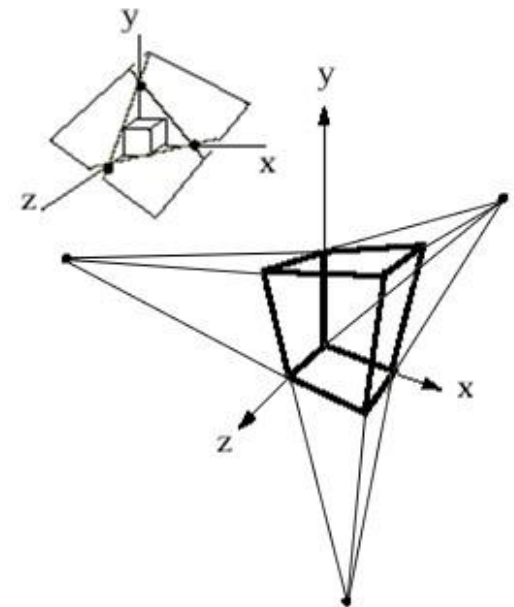
- ▶ Lines extending from edges converge to common vanishing point(s)
- ▶ For right-angled forms whose face normals are perpendicular to the  $x$ ,  $y$ ,  $z$  coordinate axes, number of vanishing points = number of principal coordinate axes intersected by projection plane



**One Point Perspective**  
( $z$ -axis vanishing point)



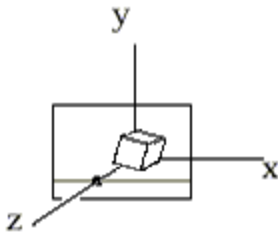
**Two Point Perspective**  
( $z$ , and  $x$ -axis vanishing points)



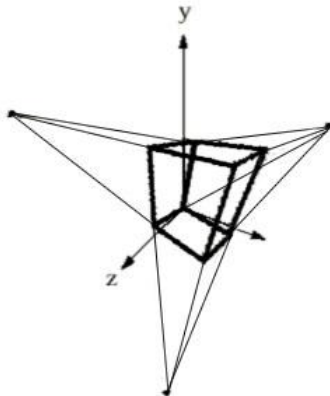
**Three Point Perspective**  
( $z$ ,  $x$ , and  $y$ -axis vanishing points)

## Vanishing Points (2/2)

- ▶ What happens if same form is turned so its face normals are *not* perpendicular to  $x$ ,  $y$ ,  $z$  coordinate axes?



*Unprojected cube depicted here  
with parallel projection*



*Perspective drawing  
of the rotated cube*

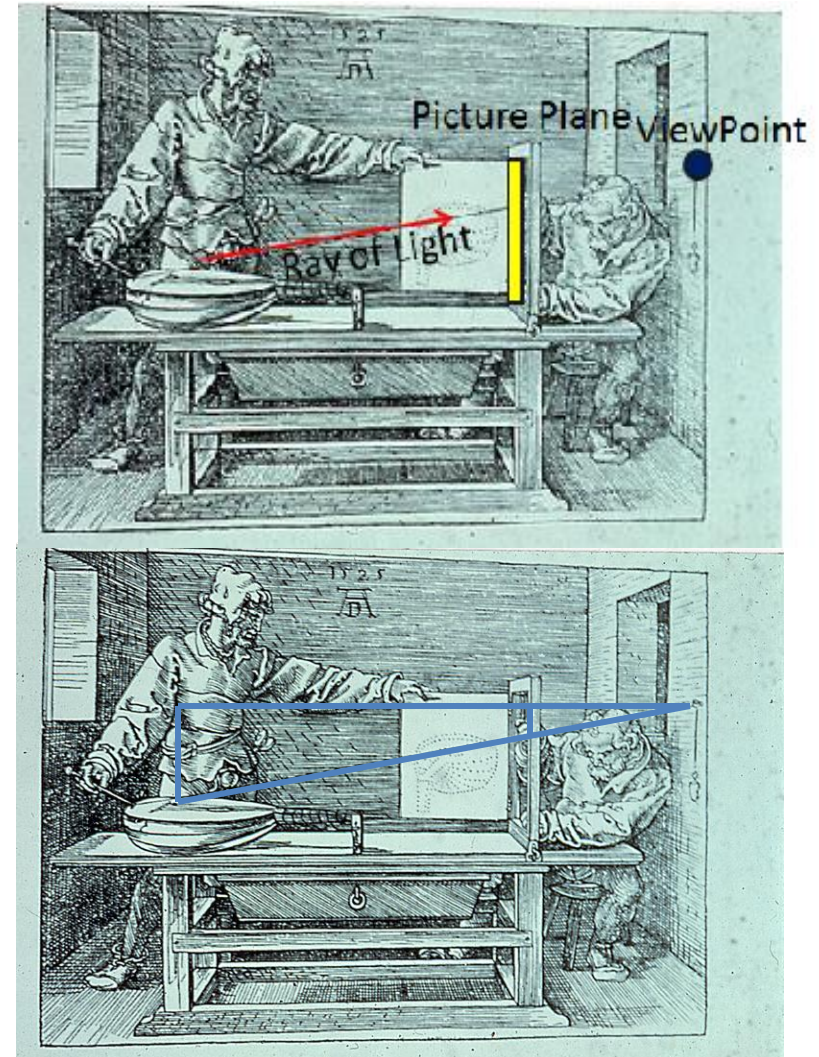
New viewing situation: cube is rotated, face normals no longer perpendicular to any principal axes

Although projection plane only intersects one axis ( $z$ ), three vanishing points created

Can still achieve final results identical to previous situation in which projection plane intersected all three axes

# The Single Viewpoint

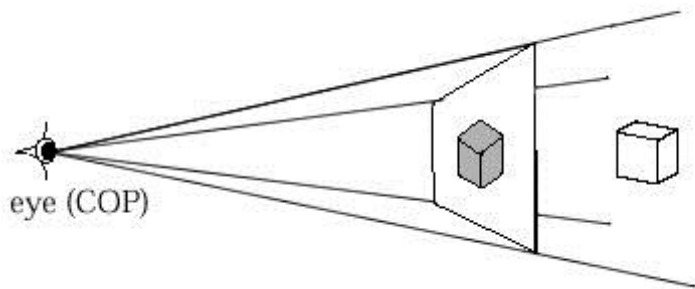
- ▶ Art employs the vanishing point idea while computer graphics uses the view point concept, where your view point is the location of the virtual camera
- ▶ Rays of light reflecting off of an object converge to the point of the viewer's eye
- ▶ Lines representing light intersect the picture plane thus allowing points in a scene to be projected along the path of light to the picture plane (Basis for Ray Tracing)
- ▶ Concept of similar triangles described earlier applies here



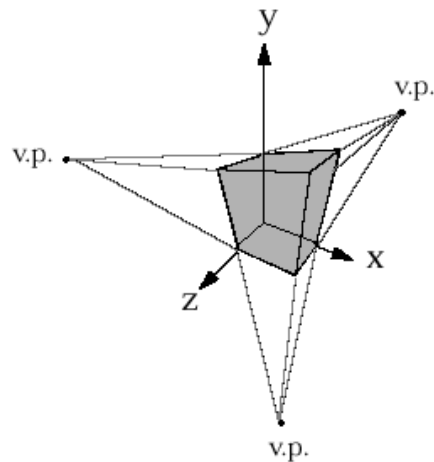


# Vanishing Points and the View Point (1/4)

- ▶ We've seen two pyramid geometries for understanding perspective projection:



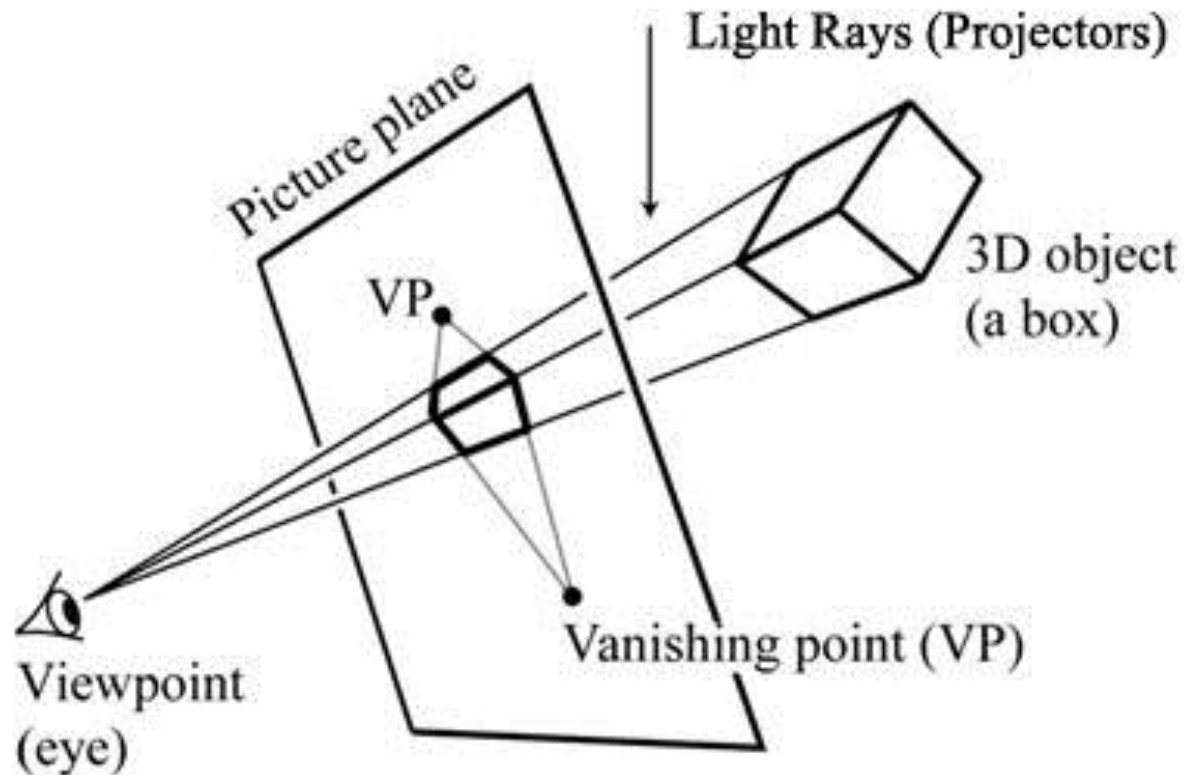
1. perspective image is intersection of a plane with light rays from object to eye (COP)



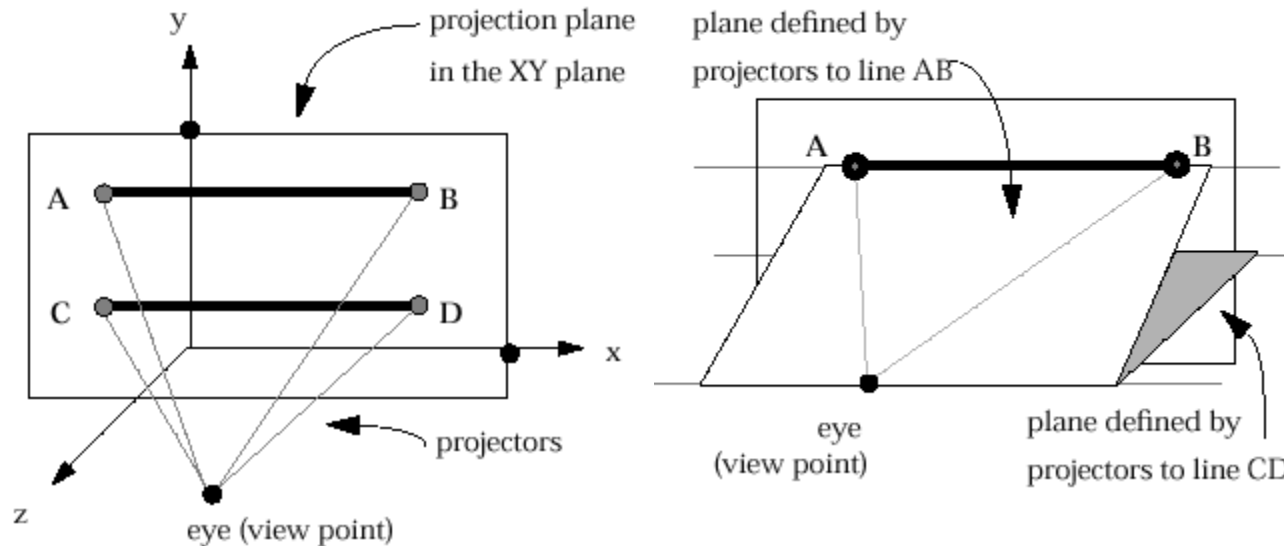
2. perspective image is result of foreshortening due to convergence of some parallel lines toward vanishing points

## Vanishing Points and the View Point (1/4)

- ▶ We can combine the two:

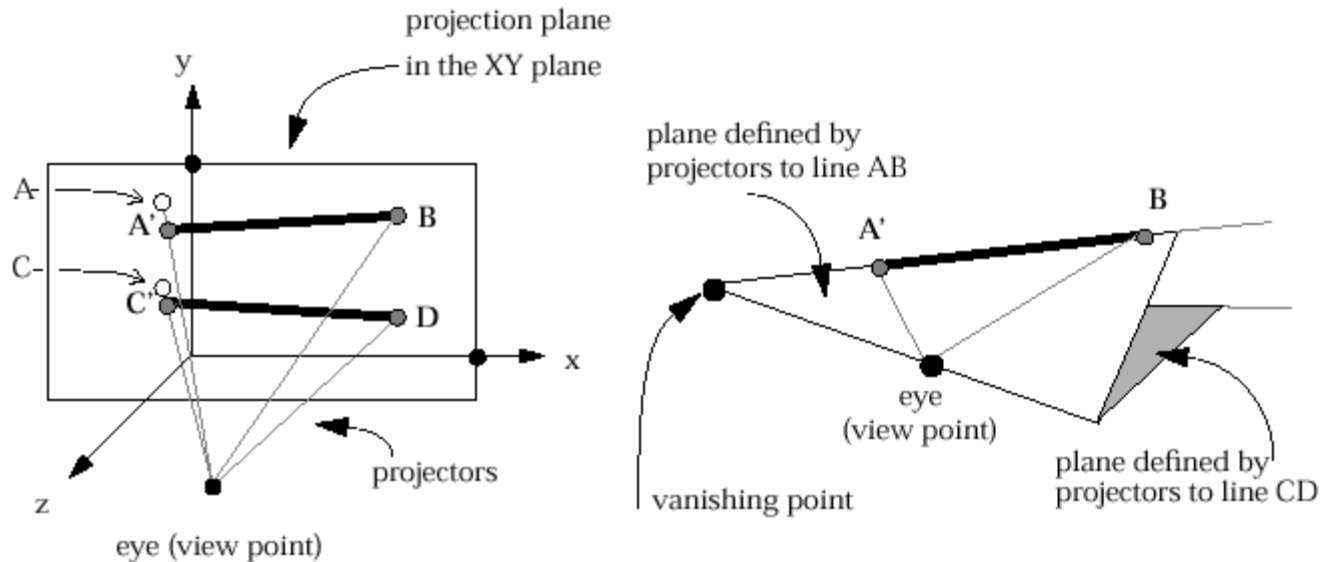


## Vanishing Points and the View Point (3/4)



- ▶ Project parallel lines  $AB$ ,  $CD$  on  $xy$  plane
- ▶ Projectors from eye to  $AB$  and  $CD$  define two planes, which meet in a line which contains the view point, or eye
- ▶ This line **does not** intersect projection plane ( $XY$ ) because it's parallel to it. Therefore, there is no vanishing point

## Vanishing Points and the View Point (4/4)



- ▶ Lines  $AB$  and  $CD$  (this time with  $A$  and  $C$  behind the projection plane) projected on  $xy$  plane:  $A'B$  and  $C'D$
- ▶ Note:  $A'B$  not parallel to  $C'D$
- ▶ Projectors from eye to  $A'B$  and  $C'D$  define two planes which meet in a line which contains the view point
- ▶ This line **does** intersect projection plane
- ▶ Point of intersection is vanishing point



# Next Time: Projection in Computer Graphics

